Seat No.: _____ Enrolment No.______

# GUJARAT TECHNOLOGICAL UNIVERSITY

**B.E. Sem-IV Examination June- 2010**

**Subject code: 140001** **Subject Name: Mathematics-4**

**Date:15 / 06 /2010** **Time: 10.30 am – 01.30 pm**

**Total Marks: 70**

## Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed. (14)

(a) Find the value of Re (f(z)) and Im (f(z)) at the indicated point where

$f(z) = \frac{1}{1-z}$ at $7 + 2i$.

(b) Find the value of the derivative of $\frac{z-i}{z+i}$ at $i$.

(c) Find an upper bound for the absolute value of the integral $\int_C e^z dz$, where C is the line segment joining the points (0,0)and $(1, 2\sqrt{2})$.

(d) Evaluate $\oint_C \frac{dz}{z^2+1}$, where C is $|z + i| = 1$, counterclockwise.

(e) Develop $f(z) = \sin^2 z$ in a Maclaurin series and find the radius of convergence.

(f) Define : (i) Singular point (ii) Essential singularity (iii) Removable singularity (iv) Residue of a function

(g) If $f(x) = \frac{1}{x}$, find the divided differences [a,b] and [a,b,c].

**Q.2** (a) Evaluate $\int_0^1 e^{-x^2} dx$ by the Gauss integration formula with n=3. (03)

(b) Compute f (9.2) from the following values using Newton's divided difference formula. (04)

| x | 8 | 9 | 9.5 | 11.0 |
|---|---|---|---|---|
| f((x) | 2.079442 | 2.197225 | 2.251292 | 2.397895 |

(c) (i) Find the positive root of x = cosx correct to three decimal places by bisection method. (03)

(ii) Solve the following system of equations using partial pivoting by Gauss-elimination method. (04)

$$8x_2 + 2x_3 = -7$$
$$3x_1 + 5x_2 + 2x_3 = 8$$
$$6x_1 + 2x_2 + 8x_3 = 26$$

**OR**

(c) (i) Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method and hence find the other eigen value also. Verity your results by any other matrix theory. (03)

(ii) Solve the following system of equations by Gauss- seidal method. (04)

$10x_1 + x_2 + x_3 = 6$
$x_1 + 10x_2 + x_3 = 6$
$x_1 + x_2 + 10x_3 = 6$

Q.3 (a) Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below : (04)

| x | -1 | 0 | 1 | 3 |
|---|---|---|---|---|
| f(x) | 2 | 1 | 0 | -1 |

(b) Evaluate $\int_0^3 \frac{dx}{1+x}$ with n=6 by using Simpson's $\frac{3}{8}$ rule and hence calculate log2. Estimate the bound of error involved in the process. (05)

(c) Using improved Euler's method, solve $\frac{dy}{dx} + 2xy^2 = 0$ with the initial condition y(0)=1 and compute y (1) taking h = 0.2. Compare the answer with exact solution. (05)

**OR**

Q.3 (a) Find an iterative formula to find $\sqrt{N}$ (where N is a positive number) and hence find $\sqrt{5}$. (04)

(b) Compute cosh 0.56 from the following table and estimate the error. (05)

| x | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|
| cosh x | 1.127626 | 1.185465 | 1.255169 | 1.337435 |

(c) Apply Runge-Kutta method of fourth order to calculate y (0.2) given (05)

$\frac{dy}{dx}$ = x+y, y (0) =1 taking h=0.1

Q.4 (a) Find and plot all roots of $\sqrt[3]{8i}$. (03)

(b) Find out (and give reason) whether f (z) is continuous at z =0 if (03)

$$f(z) = \frac{\text{Re}(z^2)}{|z|}, \quad z \neq 0$$
$$= 0 \quad , z = 0$$

(c) Using residue theorem, evaluate $\oint_C \frac{z^2 \sin z}{4z^2 - 1}$ dz, c : | z | = 2 (04)

(d) (i) Expand f (z) = $\frac{1-e^z}{z}$ in Laurent's series about z =0 and identify the singularity. (02)

(ii) Find all solutions of sinz = 2. (02)

**OR**

Q.4 (a) Solve the equation $z^2 - (5+i)z + 8 + i = 0$. (03)

(b) Show that if f(z) is analytic in a domain D and | f(z) | = k = const. in D, then f (z) = const. in D. (03)

(c) Find all Taylor and Laurent series of f(z) = $\frac{-2z+3}{z^2 - 3z + 2}$ with center 0. (04)

(d) (i) Find the center and the radius of convergence of the power series (02)

$$\sum_{n=0}^{\infty} (n+2i)^n z^n$$

(ii) State and prove Cauchy's residue theorem. (02)

Q.5 (a) Find and sketch the image of region x ≥ 1 under the transformation $w = \frac{1}{z}$ (03)

(b) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$ (03)

(c) Evaluate $\int_C \mathrm{Re}(z^2)dz$, where C is the boundary of the square with vertices 0, i, 1 + i, 1 in the clockwise direction. (04)

(d) (i) State and prove cauchy integral theorem. (02)

(ii) Determine a and b such that $u = ax^3 + bxy$ is harmonic and find a conjugate harmonic. (02)

**OR**

Q.5 (a) Define Mobius transformation. Determine the mobius transformation that maps $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ onto $w_1 = -1$, $w_2 = -i$, $w_3 = 1$ respectively. (03)

(b) Using contour integration, show that $\int_0^{\infty} \frac{dx}{1 + x^4} = \frac{\pi}{2\sqrt{2}}$ (03)

(c) Evaluate $\oint_C \frac{e^z}{z(1-z)^3}$ dz, where C is (a) $|z| = \frac{1}{2}$ (b) $|z-1| = \frac{1}{2}$. (04)

(d) Check whether the following functions are analytic or not. (04)

(1) $f(z) = z^{\frac{5}{2}}$ (ii) $f(z) = \bar{z}$

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