 Ex. 228

F.E Sem I (O) All branches. Applied maths.

**Con. 2940-08.** **CO-5102**

**(OLD COURSE)**

(3 Hours) **[Total Marks : 100**

MASTER

**N.B.**(1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicate marks.

1. (a) Prove that $\sin^{-1} ix = 2n\pi + i \log\left(x + \sqrt{1+x^2}\right)$. 5

(b) If $y = \sin^2 x \cos^3 x$ find $y_n$. 5

(c) If $z = f(x, y)$, $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that :— 5

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$

(d) Show that the vector : 5

$$\left(\bar{a} \times \bar{b}\right) \times \left(\bar{c} \times \bar{d}\right) + (\bar{b} \times \bar{c}) \times (\bar{a} \times \bar{d}) + (\bar{c} \times \bar{a}) \times (\bar{b} \times \bar{d})$$

is a vector parallel to $\bar{d}$.

2. (a) Considering only the principal value, if $(1 + i\tan\alpha)^{1+i\tan\beta}$ is real, prove that its value is $(\sec\alpha)^{\sec^2\beta}$. 8

(b) Use De'Moivre's theorem to show that— 6

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$

(c) Show that— 6

$$\tan\left[i\log\left(\frac{a-bi}{a+bi}\right)\right] = \frac{2ab}{a^2-b^2}.$$

3. (a) For the curve $\bar{r} = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$, prove that $2(k^2 + \tau^2) = 1$. 8

(b) Prove that $\tanh^{-1}(\sin\theta) = \cosh^{-1}(\sec\theta)$ 6

(c) If $x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$, prove that :— 6

(i) $x_1 \cdot x_2 \cdot x_3 \ldots\ldots \infty = i$
(ii) $x_0 \cdot x_1 \cdot x_2 \ldots\ldots, \infty = -i$

4. (a) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots$. 8

Hence expand $\log(1 + x^2)$ in powers of x.

(b) Find the equations of the osculating plane and normal plane to the curve. 6
$x = 2t^3$, $y = 3t^2$, $z = 6t$ at $t = 1$.

(c) Examine the validity of the conditions and the conclusion of Lagrange's M.V.T. for the function $\sqrt{x^2 - 4}$ on $[2, 3]$. 6

**[TURN OVER**

 FN-Ex. 229

F.E. Sem I (old) All Br., Applied Maths 20/5/08

Con. 2940-CO-5102-08. 2

5. (a) If $u = \dfrac{x^2y^2z^2}{x^2+y^2+z^2} + \cos^{-1}\left(\dfrac{x+y+z}{\sqrt{x}+\sqrt{y}+\sqrt{z}}\right)$ 8

then find the value of—

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}.$$

(b) Find the stationary value of $xy(3 - x - y)$. 6

(c) Evaluate— $\lim\limits_{x \to 0} \left(1-x^2\right)^{1/\log(1-x)}$ 6

6. (a) If $y = \left[\log\left(x+\sqrt{1+x^2}\right)\right]^2$ 8

Prove that $(1 + x^2)\, y_{n+2} + (2n + 1)\, x\, y_{n+1} + n^2 y_n = 0$.
Hence deduce that $y_{n+2}(0) = -n^2 y_n(0)$.

(b) If $u = f\left(\dfrac{x}{y}\right) + \sqrt{x^2+y^2}$ 6

Prove that— $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sqrt{x^2+y^2}$.

(c) Apply Taylor's theorem to expand— 6
$x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$

7. (a) If $u = e^{xyz} f\left(\dfrac{xy}{z}\right)$, prove that— 8

$$x\frac{\partial u}{\partial x} + z\frac{\partial u}{\partial z} = 2xyz\,u$$

$$y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2xyz\,u$$

Hence show that $x\dfrac{\partial^2 u}{\partial z\,\partial x} = y\dfrac{\partial^2 u}{\partial z\,\partial y}$.

(b) Find $[(3\cdot82)^2 + 2(2\cdot1)^3]^{1/5}$ 6
approximately by using the theory of Approximation

(c) If $x = \tan \log y$ 6
Prove that—
$(1 + x^2)\, y_{n+1} + (2nx - 1)\, y_n + n(n - 1)\, y_{n-1} = 0$.