UG–475 BMS–11/BMC–11

B.Sc. DEGREE EXAMINATION –
JANUARY 2009.

First Year

Mathematics/Mathematics with Computer Applications

ELEMENTS OF CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. If $y = a\cos 5x + b\sin 5x$ show that $\frac{d^2y}{dx^2} + 25y = 0$.

2. Verify Euler's theorem for the function $u(x, y) = x^3 + y^3 + 3x^2y + 3xy^2$.

3. Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = p$ where α is the parameter.

4. Find the surface area of a sphere of radius a.

5. Evaluate $\int_{0}^{\pi/2} \int_{0}^{a} dr\, d\theta$.

6. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{n}$ for every $n \in N$ converges to 0.

7. Define

   (a) Monotonic sequence

   (b) Cauchy sequence.

8. Test the convergence of $\sum \sqrt{\frac{2n^2 + 3}{5n^3 + 7}}$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)\, xy_{n+1} + (n^2 + 1)\, y_n = 0$.

10. Find the maximum and minimum values of $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$.

11. Show that the radius of curvature of the curve $y^2 = a^2 \frac{(a - x)}{x}$ at $(a, 0)$ is $\frac{a}{2}$.

12. (a) Evaluate $\int_0^1 x\,(1-x)^{10}\,dx$.

(b) Prove that

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx .$$

13. Establish a reduction formula for $I_n = \int \cos^n x\,dx$ where $n \in N$ and hence find $\int_0^{\pi/2} \cos^6 x\,dx$.

14. If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty} (a_n \pm b_n) = a \pm b$.

15. (a) Test the convergence of the series

$$\frac{3}{1^2\cdot 2^2} + \frac{5}{2^2\cdot 3^2} + \frac{7}{3^2\cdot 4^2} + \cdots\infty$$

(b) Show that a series $\sum_{n=1}^{\infty} u_n$ of positive terms either converges or diverges to ∞ but never oscillates.

16. Let Σu_n and Σv_n be the two given series of positive numbers such that $u_n < kv_n$ for every $n \in N$ where k is a positive number. Then prove that

(a) If Σv_n is convergent Σu_n is convergent

(b) If Σu_n is divergent then Σv_n is divergent.
