

Fifth Semester B.Tech. Degree Examination

Model Question Paper

(2008 Scheme)

Computer Science / Information Technology(RF)

ADVANCED MATHEMATICS AND QUEUING MODELS (RF)

Time : 3 Hours

Max.Marks: 100

Instructions : Answer *all* questions of Part – A and *one full question each* from Module I, Module II and Module III

PART-A

1. Define basic feasible solution, optimal solution and degenerate solution.
2. Rewrite in standard form for the following LPP.

$$\begin{aligned} \text{Max } z &= 2x_1 + x_2 + 4x_3 \\ \text{Subject to } &-2x_1 + 4x_2 \leq 4 \\ &x_1 + 2x_2 + x_3 \geq 5 \\ &2x_1 + 3x_3 \leq 2 \end{aligned}$$

Where $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

3. What are the three main phases of a project ?
4. Distinguish between CPM and PERT.
5. Given v_1 and v_2 in a vectorspace V , let $H = \text{Span}\{v_1, v_2\}$. Show that H is a subspace of V .
6. Define linearly independent set. If $p_1(t)=1, p_2(t)=t$ and $p_3(t)=4-t$ then prove that $\{p_1, p_2, p_3\}$ is linearly dependent.
7. Find the dimension of the subspace

$$H = \left\{ \begin{pmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{pmatrix} : a, b, c, d \text{ in } R \right\}$$
8. Write down the characteristic of waiting time distribution for the model $M/M/1/\infty/\text{fifo}$.
9. Define transient and steady states.
10. At a one man barber shop, customers arrive according to Poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 min. It is assumed that because of his excellent reputation customers were always willing to wait. What is the average number of customers in the shop and the average number of customers waiting for a hair cut.

(4x10=40 Marks)

Module I

11. Solve the following LPP

$$\begin{aligned} \text{Min } z &= 4x_1 + x_2 \\ \text{Subject to } &3x_1 + x_2 = 3 \\ &4x_1 + 3x_2 \geq 6 \\ &x_1 + 2x_2 \leq 4 \\ &\text{and } x_1, x_2 \geq 0 \end{aligned}$$

12. Construct the network for the project whose activities are given below and compute the total float, free float and independent float of each activity and hence determine the critical path and the project duration.

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration (in weeks)	3	8	12	6	3	3	8	5	3	8

Module II

13. (a) Define Orthogonal set. If $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$, $u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$ then show that $\{u_1, u_2, u_3\}$ is an orthogonal set.

(b) Let $W = \text{Span}\{x_1, x_2\}$, Where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis $\{v_1, v_2\}$ for W .

(c) Find a least-squares solution of the inconsistent system $Ax = b$ for

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

14. (a) Find an LU factorization of $A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$

(b) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$

Module III

15. In a super market the average arrival rate of customers is 5 in every 30 minutes. The arrival time it takes to list and calculate the customers purchase at the cash desk is 4.5 min. and this time is exponentially distributed.

- (a) How long will the customer expect to wait for service at the cash desk.
- (b) What is the chance that the queue length will exceed 5.
- (c) What is the probability that the cashier is working.

16. A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 min.

- (a) What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day.
- (b) If the subscriber will wait and are serviced in turn. What is the expected waiting time.

(3x20=60 Marks)