Reg. No. ....

Name ....

(2 pages)

K 5104

FINAL YEAR B.Sc. DEGREE EXAMINATION, MARCH/APRIL 2005

Part III—Group I—Mathematics

Paper IV—DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND VECTORS

Time : Three Hours | Maximum : 65 Marks

A maximum of 13 marks can be earned from each unit.

Unit I

1. Solve $(3y + 2x + 4)\,dx - (4x + 6y + 5)\,dy = 0$. (5 marks)

2. Find the orthogonal trejectories of the family of parabolas $y^2 = 4ax$. (4 marks)

3. Show that the differential equation $(x^2 - 4xy + 2y^2)\,dx + (y^2 + 4xy - 2x^2)\,dy = 0$ is exact, and hence solve it. (4 marks)

4. Solve $(D^2 - 2D + 1)\,y = e^{3x}$, where $D = \dfrac{d}{dx}$. (3 marks)

5. Solve $\dfrac{d^2y}{dx^2} + y = \cos^2 x$. (4 marks)

Unit II

6. Solve $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - 9y = \log x$. (4 marks)

7. Solve $\dfrac{dx}{dt} = x + 5y$; $\dfrac{dy}{dt} = -x - 3y$. (6 marks)

8. Find the Laplace transform of $t\,e^{-t} \sin t$. (4 marks)

9. Find :

   (a) $L^{-1}\left\{\dfrac{2s - 1}{s^2 - s}\right\}$.

   (b) $L^{-1}\left\{\dfrac{2s}{(s - 1)(s - 2)^2}\right\}$.

   (3 + 3 = 6 marks)

Turn over

Unit III

10. Define the shift operator "E" and central difference operator "δ" show that $\delta = E^{1/2} - E^{-1/2}$.

(5 marks)

11. Derive Gauss backward formula for interpolation. (5 marks)

12. Find x at $y = 1.4$ from the following data :—

Y :	1.2	2.0	2.5	3.0
X :	1.34	0.57	0.33	0.21

(4 marks)

13. Show that the n^{th} divided differences of a polynomial of n^{th} degree are constant. (6 marks)

Unit IV

14. Show that $\left[\bar{a} + \bar{b}, \bar{b} + \bar{c}, \bar{c} + \bar{a}\right] = 2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$. (4 marks)

15. If $\bar{F} = (x + y + 1)\bar{i} + \bar{j} - (x + y)\bar{k}$, show that $\bar{F} \cdot \text{curl}\,\bar{F} = 0$. (4 marks)

16. Determine the constant "a" so that the vector $\bar{f} = (x + 3y)\bar{i} + (y - 2z)\bar{j} + (x + az)\bar{k}$ is solenoidal.

(4 marks)

17. If $\phi(x, y, z) = 3x^2y - y^3z^2$, find grad ϕ at $(1, -2, -1)$. (5 marks)

18. If $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ are coplanar vectors, show that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = 0$. (3 marks)

Unit V

19. Evaluate $\int_C \bar{r} \cdot d\bar{r}$ where C is the helical path $x = \cos t, y = \sin t, z = t$ joining the points determined by $t = 0$ and $t = \pi/4$.

(5 marks)

20. Find $\iint_S \left[z\bar{i} + x\bar{j} + y\bar{k}\right] \cdot \bar{n}\, ds$ where "S" is the quadrant of the circle $x^2 + y^2 = 1$ between the positive parts of the axes.

(5 marks)

21. State Gauss Divergence theorem. (3 marks)

22. State Stoke's theorem. Verify the theorem for the function $\bar{F} = (2x - y)\bar{i} - yz^2\bar{j} - y^2z\bar{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

 (2 + 5 = 7 marks)