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Your Roll No. ....

M.A. / Winter Semester **A**

ECONOMICS

Course 106— Topics in Economic Theory

(Admissions of 1999 and onwards)

Time : 21/2 hours *Maximum Marks* : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any three of the 4 questions given below. Each question carries a total of 231/3 marks. Marks for each part of a question are indicated in parentheses.

(1). Consider a Markov Process on a finite state space S (with $|S| = k$), with transition probability matrix M. Use the norm $||y||_1 \equiv \sum_{i=1}^{k} |y_i|$ on $\Re^k$. Suppose there is a state j_0 and $\epsilon > 0$ s.t. for all states $i \in S$, $M_{ij_0} \geq \epsilon$.

Let δ_{j,j_0} be the indicator variable that equals 1 if $j = j_0$, and equals 0 if the state $j \neq j_0$.

(A). Let $y \in \Re^k$ s.t. $\sum_i y_i = 0$. Show that for all states j,

$$|(yM)_j| \leq \sum_{i \in S} |y_i|(M_{ij} - \epsilon\delta_{j,j_0})$$

(B). Infer from the above that

$$||yM||_1 \leq (1 - \epsilon)||y||_1$$

(C). Notice that if ϕ and ψ are probability vectors, then (B) holds with y replaced by $\phi - \psi$, as the coordinates of this add up to 0. Now let μ be a probability vector, and write $\mu_n = \mu M^n$. By repeatedly iterating the result in (B), show that, with $n > m$,

$$||\mu_n - \mu_m||_1 \leq (1 - \epsilon)^m ||\mu_{n-m} - \mu||_1 \leq C(1 - \epsilon)^m$$

for some $C > 0$.

(D). Hence the sequence $(\mu_n)_{n=0}^{\infty}$ is Cauchy, and converges to a probability vector π. Show that $\pi = \pi M$, i.e., that π is stationary.

$(6, 6, 6, 5\frac{1}{3})$

(2). (A). Let (S, ρ) be a complete metric space and let the function $f : S \to S$ satisfy $\rho(f(x), f(y)) < \rho(x, y)$ for all distinct $x, y \in S$. Let $f^m(x) \equiv$

$f(f(\ldots(f(x))))$ be the function obtained by applying f $m-$ times. Fix $x \in S$. Show that then the sequence of distances $(\rho(f^{m+1}(x), f^m(x)))_{m=1}^{\infty}$ is a convergent sequence.

(2). (B). Let (S, ρ) be a complete metric space, and let $(\Phi_m)_{m=1}^{\infty}$ be a sequence of uniformly strict contractions with modulus $\lambda, 0 < \lambda < 1$, from S into S. Let (x_m) be the corresponding unique fixed points of (Φ_m) (due to Banach's Theorem). Suppose there exists a function $\Phi : S \to S$ such that

$$\sup\{\rho(\Phi_m(x), \Phi(x)) | x \in S\} \to 0 \ as \ m \to \infty$$

Show that then Φ is a uniformly strict contraction with unique fixed point $x^* = \lim x_m$. Hint: Estimate the distance $\rho(\Phi(x), \Phi(y))$ by breaking it up into distances about which you have information regarding the sup assumption above, or about contractions.

$(10, 13\frac{1}{3})$

(3). Let S be a state space and $\mathcal{D}$ be the set of all prospects on it (all real valued functions on S taking on a finite number of values).

(A). Suppose a decisionmaker's (DM's) preference relation $\succeq$ on $\mathcal{D}$ is a weak order and satisfies monotonicity. Suppose also that for every prospect x, there exists a certainty equivalent $CE(x)$. Show that then CE represents $\succeq$.

(B). Suppose in addition (to the assumptions in (A)) that $\succeq$ satisfies additivity. Show that then for every pair of prospects x, y, $CE(x + y) = CE(x) + CE(y)$.

(C). Suppose a coin is tossed, giving H or T. Suppose $\succeq$ is a weak order, and that all outcomes α, β, we have $\alpha_H\beta \sim \beta_H\alpha$. Assume risk aversion in the sense that there exist outcomes γ, β, with $\gamma > \beta$ s.t. $CE(\gamma_H\beta) < (\beta + \gamma)/2$. Show that the preference contains a Dutch Book.

$(7, 7, 9\frac{1}{3})$

(4). Consider the following infinite-horizon model of the market for a commodity. Time is discrete $(t = 0, 1, 2, \ldots)$. Harvests $(W_t)_0^{\infty}$ are i.i.d. according to the density ϕ on $S = [a, \infty)$, $a > 0$. Final consumers' demand is $D(p)$, if the market price is p in any period, and the inverse demand function P is strictly decreasing and continuous. I_t units purchased by speculators at time t yields αI_t units at time $t + 1$, $(\alpha \in (0, 1))$. Risk-neutral speculators' expected profits are $\mathbf{E}_t p_{t+1} \alpha I_t - p_t I_t$, where p_t, p_{t+1} are market prices at times $t, t + 1$, and $\mathbf{E}_t$ refers to expectation conditional on information available at time t.

The supply of the commodity at time 0 is given, and equals $X_0 \in S$. Note that supply at time t, $X_t = \alpha I_{t-1} + W_t$ and demand $= D(p_t) + I_t$.

An equilibrium is a sequence $(I_t, p_t, X_t)_{t \geq 0}$ of random variables such that there is a function $p^* : S \to (0, \infty)$ with $p_t = p^*(X_t), \forall t$, and the following conditions are satisfied:

(i) (no arbitrage): $\alpha \mathbf{E}_t p_{t+1} - p_t \leq 0, \forall t$.

(ii) Profit Maximization by Speculators: If $\alpha \mathbf{E}_t p_{t+1} - p_t < 0$, then $I_t = 0$.
(iii) Market clearing: $X_t \equiv \alpha I_{t-1} + W_t = D(p_t) + I_t$.

(2.1). Show that there is a unique function $p^* : S \to (0, \infty)$ that solves

$$p^*(x) = \max\left\{\alpha \int p^*(\alpha I(x) + z)\phi(z)dz, P(x)\right\}, \forall x \in S$$

(Hint: Use Blackwell's Theorem and Banach's Contraction Mapping Theorem, taking their conclusions as given).

(2.2). Show that the p^* defined above can serve as the price functional required in the definition of an equilibrium.

$(18, 5\frac{1}{3})$