

This question paper contains 4+2 printed pages]

Your Roll No . . .

5186

**B.Sc. Prog./II**

**J**

**MA-201—MATHEMATICS—I**

**(Calculus and Geometry)**

**(For Physical Sciences/Applied Sciences)**

**(Admissions of 2008 and onwards)**

**Time 3 Hours**

**Maximum Marks . 112**

*(Write your Roll No on the top immediately on receipt of this question paper )*

**All questions are compulsory**

**Attempt any two parts from each question except**

**Question No 5 In questions No 5 attempt any one**

**Section I**

1 (a) (i) Represent geometrically the set

$$\{z \mid |z - 1| > |z + 1|\} \quad 5\frac{1}{2}$$

(ii) Find centre and radius of the circle whose equation is

$$|z - 1| = 4|z + 1|$$

Check if the origin lies inside the circle 7

**P.T O**

- (b) (i) Solve the equation .

$$x^4 - 2x^3 - 21x^2 - 22x + 40 = 0,$$

the sum of the two roots being equal to  
sum of the other two 7

- (ii) If  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the equation

$$x^3 - \alpha x^2 + bx - c = 0, \text{ find the value of}$$

$$\sum \frac{\alpha^2}{\beta\gamma}. \quad 5\frac{1}{2}$$

- (c) (i) Prove that :

$$(1+i)^n + (1-i)^n = 2^{n/2+1} \cos(n\pi/4) \quad 5\frac{1}{2}$$

- (ii) Solve the equation

$$z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0 \quad 7$$

## Section II

- 2 (a) State intermediate value theorem and show that if the continuity hypothesis of the theorem is dropped, then its conclusion may fail to hold 10

- (b) Show that the function  $f$  defined as

$$f(x) = |x| + |x - 1|$$

is continuous at  $x = 0$ , and  $x = 1$  But is not derivable at these points 10

- (c) Define uniform continuity of a function. Give an example of a continuous function which is not uniformly continuous 10

- 3 (a) Find the asymptotes of the curve

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0. \quad 10$$

- (b) (i) Determine the position and nature of double points on the curve

$$x^3 + x^2 + y^2 - x - 4y + 3 = 0$$

- (ii) Trace the polar curve

$$r = a(1 + \cos \theta) \quad 5+5$$

P T O

- (c) Trace the curve 10

$$y(1 - x^2) = x^2$$

- 4 (a) Obtain the reduction formula for

$$\int \sin^n x \, dx,$$

$n$  being a positive integer and hence evaluate

$$\int_0^{\pi/2} \sin^6 x \, dx \quad 10$$

- (b) Find the area of the parabola  $y^2 = 4ax$

bounded by its latus rectum Also find the

volume of the solid obtained by rotating this

area about  $x$ -axis 10

- (c) Find the arc length of the cycloid

$$x = a(\theta + \sin \theta), y = a(1 + \cos \theta), -\pi \leq \theta \leq \pi \quad 10$$

Section III

- 5 Identify and sketch the graph of the conic

$$4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$$

by rotating co-ordinate axes 14

Or

Find the nature of the following conic and trace it completely giving essential details

$$x^2 + xy + y^2 - x + 4y + 3 = 0 \quad 14$$

- 6 (a) A particle moves along the curve  $x = t^3 + 1$ ,  
 $y = t^2$ ,  $z = 2t + 5$ , where  $t$  is the time Find  
the component of the velocity and acceleration  
at  $t = 1$  in the direction  $\hat{i} + \hat{j} + 3\hat{k}$  6½

- (b) Show that

$$\nabla^2 \left( \frac{1}{r} \right) = 0 \quad 6½$$

P T O

- (c) For what value of the component  $a$  will the vector

$$\vec{A} = (axy - z^3)\hat{i} + (a - 2)x^2\hat{j} + (1 - a)xz^2\hat{k}$$

have its curl identically equal to zero ? 6½