This question paper contains 4 printed pages

**5194**

Roll No

# B. Sc. Prog. /II

# OPERATIONAL RESEARCH

## OR – 201 : - Optimization

(Admissions of 2005 and onwards)

J

Time 3 hours

Maximum Marks 112

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt any **five** questions

**1(a)** Discuss applications of Operations Research and advantages of Operations Research approach in decision making

**(b)** A company has two grade of inspectors, $I$ and $II$, who are to be assigned for a quality control inspection It is required that at least 2000 items be inspected per 8 hour day Grade $I$ inspector can check items at the rate of 50 per hour with the accuracy of 97% while Grade $II$ inspector can check items at the rate of 40 per hour with the accuracy of 95% The wage rate of Grade $I$ inspector is Rs 4 50 per hour and that of Grade $II$ inspector is Rs 2 50 per hour Each time an error is made by any inspector, the cost to the company is one rupee The company has 10 Grade $I$ and 5 Grade $II$ inspectors available with it Formulate a linear programming problem to minimize the total cost of inspection and solve it graphically

**(10, $12\frac{1}{2}$)**

**2(a)** Define a convex set Is the intersection of any finite number of convex sets necessarily a convex set ? Justify

**(b)** If $S$ and $T$ are any two convex sets in $R^n$ then show that for all scalars $\alpha, \beta$, the set $\alpha S + \beta T$ is also a convex set

**(11, $11\frac{1}{2}$)**

**3(a)** Find all basic feasible solutions for the system

$$x_1 + 4x_2 + x_3 = 8$$
$$x_1 + 2x_2 + x_4 = 4$$
$$x_1, x_2, x_3, x_4 \geq 0$$

**(b)** Are the following constraints consistent? Use Simplex method to check the same

$$2x_1 - 3x_2 \geq 2$$
$$-x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

**($11\frac{1}{2}$, 11)**

200

**4(a)** Solve the linear programming problem

Minimize $z = 2x_1 - x_2 + 2x_3$

subject to $-x_1 + x_2 + x_3 = 4$

$-x_1 + x_2 - x_3 \le 6$

$x_1 \le 0, x_2 \ge 0$

$x_3$ unrestricted in sign

**(b)** Prove that if all $z_j - c_j \ge 0$ then the current basic feasible solution $X_B$ of a linear programming problem is an optimal solution

$(11\frac{1}{2}, 11)$

**5(a)** Consider the linear programming problem

Maximize $z = c^T x$

subject to $Ax \le b$,

$x \ge 0$

Write the dual problem. State and prove the complementary slackness theorem

**(b)** Consider the linear programming problem

Maximize $z = 5x_1 + 4x_2$

subject to $x_1 - 2x_2 \le 1$,

$x_1 + 2x_2 \ge 3$,

$x_1, x_2 \ge 0$

Use duality relationships to show that the above problem has an unbounded solution

$(11\frac{1}{2}, 11)$

**6(a)** Explain any one method to obtain a basic feasible solution of the transportation problem. Solve the following transportation problem for minimizing the total cost

| Origins | Destinations | | | Availability |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | |
| $O_1$ | 0 | 2 | 0 | 70 |
| $O_2$ | 1 | 4 | 0 | 30 |
| $O_3$ | 0 | 2 | 4 | 50 |
| Requirements | 70 | 50 | 30 | |

**(b)** Consider the problem of assigning five operators to five machines with the following assignment cost matrix

| Operators | I | II | III | IV | V |
|---|---|---|---|---|---|
| | **Machines** | | | | |
| $A$ | 10 | 5 | 13 | 15 | 16 |
| $B$ | 3 | 9 | 18 | 3 | 6 |
| $C$ | 10 | 7 | 2 | 2 | 2 |
| $D$ | 5 | 11 | 9 | 7 | 12 |
| $E$ | 7 | 9 | 10 | 4 | 12 |

Assign the operators to the machines so that the total cost is minimized

**(12½, 10)**

**7(a)** Solve the integer linear programming problem

Maximize $z = 7x_1 + 9x_2$

subject to $-x_1 + 3x_2 < 6$

$7x_1 + x_2 \leq 35$

$x_1 \leq 7$

$x_2 < 7$

$x_1, x_2 \geq 0$ and are integers

The non-integer optimal solution of the problem is $x_1 = \frac{9}{2}$, $x_2 = \frac{7}{2}$, $z = 63$

(b) The optimal simplex table of the linear programming problem

Maximize $z = 3x_1 + 4x_2 + x_3 + 7x_4$

subject to $8x_1 + 3x_2 + 4x_3 + x_4 \le 7$

$2x_1 + 6x_2 + x_3 + 5x_4 \le 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 \le 8$

$x_1, x_2, x_3, x_4 \ge 0$

is given as follows

| | | $c_j$ | 3 | 4 | 1 | 7 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | $y_B$ | $b$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $s_1$ | $s_2$ | $s_3$ |
| 3 | $x_1$ | 16/19 | 1 | 9/38 | 1/2 | 0 | 5/38 | -1/38 | 0 |
| 7 | $x_4$ | 5/19 | 0 | 21/19 | 0 | 1 | -1/19 | 4/19 | 0 |
| 0 | $s_3$ | 126/19 | 0 | 59/38 | 9/2 | 0 | -1/38 | -15/38 | 1 |
| | | $z_j - c_j$ | 0 | 169/38 | 1/2 | 0 | 29/38 | 53/38 | 0 |

Discuss the effect on the above optimal solution by changing the resource vector $b = (7, 3, 8)$ to $\hat{b} = (13, 3, 8)$. If the new solution is not feasible then obtain a feasible solution using dual simplex method.

(11 $1\frac{1}{2}$)