

# Thapar Institute of Engineering & Technology

End semester Examination Dec. 09, 2006

Time: 3 hrs

Subject: Basic Mathematics ( MA-502)

MM : 45

Note: Attempt any five questions. Do all questions in sequence.

- Q1(a) If the sum of the roots of equation  $x^2 - px + q = 0$  be  $m$  times their difference, then prove that  $p^2(m^2 - 1) = 4m^2q$ . 3
- (b) Solve the equation  $x(x + 3) + 3\sqrt{(2x^2 + 6x + 5)} = 25$  3
- (c) Find the coefficients of  $x^{10}$  and  $x^6$  in the expansion of  $\left(2x^2 - \frac{3}{x}\right)^{11}$ . 3
- Q2(a) Show that the coefficient of mid term of  $(1 + x)^{2n}$  is equal to the sum of the coefficient of the two mid terms of  $((1 + x)^{2n-1})$ . 3
- (b) If  $(x + iy)^{1/3} = a + ib$ , Prove that  $4(a^2 - b^2) = \frac{x}{a} + \frac{y}{b}$ . 2.5
- (c) Find the square root of  $-2 + 2\sqrt{3}i$ . 3.5
- Q3(a) If  $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$ , in which  $a, b$  and  $c$  are different, show that  $abc = 1$ . 3.5
- (b) Apply the Cramer's rule to solve the equations 2.5  
 $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$
- (c) Find  $n$  if  ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22 : 7$ . 3
- Q4(a) Find the intervals on which  $f(x) = -x^3 + 12x + 5, -3 \leq x \leq 3$  is increasing and decreasing. Where does the function assume extreme values and what are these values? 2.5
- (b) Graph the function  $y = x^{2/3} \left(\frac{5}{2} - x\right)$  4.5
- (c) In how many ways four books of physics three books of chemistry and two books of mathematics can be arrange in a shelf so that books of the same subjects are put together. 2
- Q5(a) Evaluate the integrals (i)  $\int_0^1 \frac{x^2}{\sqrt{(3x+1)}} dx$  (ii)  $\int \frac{x^3}{(x-2)(x+1)(2x+3)} dx$  2.5+  
3.5
- (b) Find the general solution of the following differential equation 3  
 $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$
- Q6(a) Find the solution of  $2y'' - 4y' + 8y = 0, y(0) = 1, y'(0) = 1$  4.5
- (b) The vertices of a quadrilateral are  $(6,3), (-3,5), (4,-2)$  and  $(x,3x)$  denoted by  $A, B, C$  and  $D$  respectively. Find the values of  $x$  so that the area of triangle  $ABC$  is double the area of triangle  $DBC$ . 4.5