

Paper X — THEORY OF EQUATIONS AND  
NUMERICAL ANALYSIS

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(For those who joined in July 2003 and after)

Time : Three hours

Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. If one root of the equation  $2x^3 - 11x^2 + 38x - 39 = 0$  is  $2 - 3i$ , find the other roots.
2. Solve  $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$ , given that it has two pairs of equal roots.
3. Transform the equation  $x^3 + px^2 + qx + r = 0$  into another, locking the second degree term.
4. Show that the equation  $x^3 + qx + r = 0$  will have one root twice another if  $343r^2 + 36q^3 = 0$ .
5. Show that  $x^3 + 3x - 1 = 0$  has only one real root and calculate it correct to two places of decimals.
6. Solve  $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$ .

7. Find a real root of the equation  $\cos x = 3x - 1$  correct to 4 decimal places by iteration method.

8. Solve the following system of equations, using Gauss-Jordan method

$$x - y + z = 1, -3x + 2y - 3z = -6, 2x - 5y + 4z = 5.$$

9. Prove that  $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{E e^x}{\Delta^2 e^x} = e^x$ , taking  $h$  as the interval of differencing.

10. By dividing the range into ten equal parts, evaluate  $\int_0^\pi \sin x dx$  by Trapezoidal rule. Verify your answer with actual integration.

11. Find the missing term in the following :

$x:$	1	2	3	4	5	6	7
$y:$	2	4	8	-	32	64	128

12. Solve  $u_n - 2u_{n-1} + u_{n-2} = 0$ .

**SECTION B — (6 × 10 = 60 marks)**

Answer any SIX questions.

13. Solve the equation  $x \tan x = -1$  by Regula Falsi method starting with  $a = 2.5$  and  $b = 3$  correct to 3 decimal places.

14. Solve, by Gauss-Seidel method the following system

$$28x + 4y - 2 = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35.$$

15. Using the following table, apply Gauss forward formula to get  $f(3.75)$

$x:$	2.5	3.0	3.5	4.0	4.5	5.0
$f(x):$	24.145	22.043	20.225	18.644	17.262	16.047

16. Use Stirling's formula to get  $\tan 89^\circ 26'$  from the table

$x:$	$89^\circ 21'$	$89^\circ 23'$	$89^\circ 25'$	$89^\circ 27'$	$89^\circ 29'$
$\tan x:$	88.14	92.91	98.22	104.17	110.90

17. Find a cubic polynomial of  $x$  given that

$x:$	0	1	2	5
$f(x):$	2	3	12	147

18. Find the age corresponding to the annuity value 13.6 given the table :

Age ( $x$ ):	30	35	40	45	50
Annuity value ( $y$ ):	15.9	14.9	14.1	13.3	12.5

19. Find the values of  ${}_{y,z} \Delta^2 x^2$  and  ${}_{y,z} \Delta^2 x^3$  and hence evaluate  ${}_{y,z} \Delta^2 (ax + b) ((x + d))$  and  ${}_{y,z} \Delta^2 (ax + b) ((x + d) (lx + f))$ .

20. Evaluate  $I = \int_4^{5.2} \log_e x \, dx$  using (a) Simpson's rule and (b) Weddle's rule.

21. Solve  $y_{n+2} - 4y_{n+1} + 3y_n = 2^n + 3^n + 7$ .

22. Solve  $\Delta u_x + \Delta^2 u_x = \cos x$ .