(b) Find the equation of the plane which passes through the point (–1, 3, 2) and perpendicular to the two planes

$$x + 2y + 2z = 5 \;;$$

$$3x + 3y + 2z = 8.$$

8. (a) Find the image of the point (2, 3, 4) in the plane

$$x - 2y + 5z = 6.$$

(b) Show that the lines :

$$\frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4}$$

and $$\frac{x-1}{5} = \frac{y-2}{8} = \frac{z-3}{-7}$$

are coplanar. Find also, the equation of the plane containing them.

9. (a) Find the equation of the sphere passing through the four points (2, 3, 1), (5, –1, 2), (2, 5, 3) and (4, 3, –1)



Register Number :

Name of the Candidate :

**5 2 3 3**

# B.Sc. DEGREE EXAMINATION, 2008

**( MATHEMATICS )**

( SECOND YEAR )

**( PART - III - A - MAIN )**

( PAPER - III )

## 650. ALGEBRA AND SOLID GEOMETRY

*( Including Lateral Entry )*

December ] [ Time : 3 Hours

Maximum : 100 Marks

*Answer any FIVE questions.*

*All questions carry equal marks.*

(5 × 20 = 100)

1. (a) Find the equation with rational co-efficients whose roots are

$$4\sqrt{3}\;,\; 5 + 2\sqrt{-1}.$$

(b) Solve the equation

$$3x^3 - 4x^2 + x + 88 = 0$$

which has a root $2 - \sqrt{-7}$.

**Turn over**



http://www.howtoexam.com

(c) Solve the equation

$$x^3 - 12x^2 + 39x - 28 = 0$$

whose roots are in arithematical progression.

2. (a) If $\alpha$, $\beta$, $\gamma$ are the roots of the equation

$$x^3 + px^2 + qx + r = 0,$$

find the equation whose roots are

$$\alpha + \beta, \ \beta + \gamma, \ \gamma + \alpha.$$

(b) Solve the reciprocal equation

$$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0.$$

3. (a) State and prove Lagrange's theorem and deduce Fermat's theorem.

(b) Show that $n^5 - n$ is divisible by 30.

4. (a) Prove that a non void subset H of a group G is a subgroup, if and only if,

$$a, b \in H \Rightarrow ab^{-1} \in H.$$

(b) State and prove the fundamental theorem of group homomorphisms.





5. (a) Show that the set of all complex numbers of the form $a + ib$ where $a$ and $b$ are integers is a commutative ring.

(b) Show that a finite integral domain is a field.

6. (a) If A and B are normal subgroup of a group G, prove that $A \cap B$ is also a normal subgroup of G.

(b) If H and K are finite-subgroups of a group G of orders O(H) and O(K) respectively, prove that

$$O\,(HK) = \frac{O(H) - O(K)}{O(H \cap K)}$$

(c) Show that every field is an integral domain.

7. (a) Show that (1, –1, 1) , (5, –5, 4), (5, 0, 8) and (1, 4, 5) are the vertices of a rhombus.

**Turn over**

(b) Show that the plane

$$2x - y - 2z = 16$$

touches the sphere

$$x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$$

and find the point of contact.

10. (a) Show that

$$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z = 20$$

represents a cone and find its vertex.

(a) Find the equation of the right circular one whose vertex is at the origin, whose axis is the line

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

and which has a vertical angle of 60°.

5

(b) Show that the plane

$$2x - y - 2z = 16$$

touches the sphere

$$x^2 + y^2 + z^2 - 4x + 2y + 2z - 3 = 0$$

and find the point of contact.

10. (a) Show that

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